Algorithms for Aircraft Parameter Estimation Accounting for Process and Measurement Noise

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A comparison is made of four algorithms for parameter estimation in linear and nonlinear systems accounting for both process and measurement noise using two different approaches: direct approach and filtering approach. In the direct approach, the iterative Gauss-Newton method incorporating a suitable state estimator is used to estimate the unknown parameters by optimization of the likelihood function. For the state estimation both time-varying and steady-state filters are used. In the filtering approach, the unknown parameters are estimated as augmented states using the extended Kalman filter. The various algorithms are used to estimate from simulated as well as flight-test data the aircraft dimensional and nondimensional derivatives. Three model postulates, one linear and two nonlinear, are employed for this purpose. The parameter-estimation results indicate that the Gauss-Newton method with a steady-state filter, found to be adequate for the typical aircraft-estimation examples considered in the paper, is generally preferable. In the event that it becomes necessary to incorporate a time-varying filter, the filtering approach appears to be a viable alternative. Different aspects, such as convergence, computational time, parameter estimates, and their accuracies are evaluated for each of the four estimation algorithms. A general set of conclusions has been drawn.

Nomenclature

A = state matrix of linearized system
a_x, a_y, a_z = accelerations along x, y, z body axes, m/s^2
b_x, b_y = bias parameters of state and observation equations
C = observation matrix of linearized system
C_L, C_D = coefficients of lift and drag
C_m = coefficient of pitching moment
C_X, C_Z = coefficients of longitudinal and vertical force
$C_{L(\cdot)}, C_{D(\cdot)}, C_{m(\cdot)}, C_{X(\cdot)}, C_{Z(\cdot)}$ = nondimensional derivatives
$\bar{c}$ = reference chord, m
e^j = jth unit vector
F = state noise matrix
F_e = net thrust, N
$f[\cdot]$ = vector of system state functions
G = measurement noise matrix
g = acceleration due to gravity, m/s^2
$g[\cdot]$ = vector of system observation functions
I_y = moment of inertia about lateral axis, kg-m^2
J = cost function
K = filter-gain matrix
k = discrete-time index
$L_{(\cdot)}, M_{(\cdot)}, N_{(\cdot)}$ = dimensional moment derivatives
m = aircraft mass, kg
m = number of observation (output) variables
N = number of data points
n = number of state variables
P = covariance matrix of the state error
p, q, r = roll, pitch, and yaw rates, rad/s
$\dot{p}, \dot{q}, \dot{r}$ = roll, pitch, and yaw accelerations, rad/s^2
$\bar{q}$ = dynamic pressure, N/m^2
R = covariance matrix of residuals (innovations)
r_k = kth diagonal element of R^{-1}
S = reference area, m^2
t = time, s
u = control input vector
u, v, w = velocity components along x, y, and z body axes, m/s
V = airspeed, m/s
v = measurement noise vector
w = state noise vector
x = state vector
$X_{(\cdot)}, Y_{(\cdot)}, Z_{(\cdot)}$ = dimensional force derivatives
y = observation vector
z = measurement vector
α = angle of attack, rad
β = vector of unknown system coefficients
$\delta_a, \delta_e, \delta_r$ = aileron, elevator, and rudder deflections, rad
δx = perturbation in x
$\delta\Theta$ = perturbation in Θ
Δt = sampling time, s
Δu = zero shift in u
Δz = zero shift in z
$\Delta\Theta$ = vector of parameter increments
θ = pitch angle, rad
Θ = vector of unknown parameters
ρ = density of air, kg/m^3
σ_T = tilt angle of the engines, rad
Φ = state transition matrix

Superscripts

T = transpose
-1 = inverse
$\sim$ = predicted estimates
$\wedge$ = corrected estimates

Subscripts

a = augmented variables
i, j = general indices
k = discrete-time index
m = measured variables
p = perturbed variables
0 = initial conditions

Received March 25, 1988; revision received Sept. 24, 1988.

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Introduction

APPLICATIONS of the parameter-estimation method accounting for measurement noise only, i.e., output error method, to extract aircraft derivatives from flight-test data based on linear model postulates have been highly successful in the past.[1-3] This success, combined with the fact that the linear filtering theory provides an optimal solution to state estimation in linear systems,[4,5] had paved the way to the use of filter error methods for linear systems.[6-12] Such methods accounting for both process and measurement noise, although essentially more complex, are not only necessary to analyze flight-test data from flights in turbulent atmosphere (process noise), but can even yield improved estimation results for flight data in a seemingly steady atmosphere.

The recent advances in aircraft parameter estimation indicate that the output error method has been successfully extended to general nonlinear systems[13-15] and that significant improvements in the estimation results are possible by considering nonlinear model postulates in the estimation.[16,17] Although extension of the filter error methods to nonlinear systems is a natural further step, and the use of an approximate filter based on a system linearization has been suggested, there are not any such applications to estimate aircraft derivatives reported in the aircraft-identification literature known to the authors. The extended Kalman filter has been used in the past for a related purpose of aircraft state estimation, often called the flight-path reconstruction or kinematic consistency checking of flight data.[18-20] In this case, however, the fixed model structure is known a priori, and information about the noise covariances can be obtained from laboratory calibrations of the measurement sensors used. As such, it constitutes a different problem than the one addressed in this paper, which is of a more general nature.

The algorithm hitherto used for parameter estimation in linear systems with unknown noise covariances[7,12] has been very recently extended to nonlinear systems.[21] This extension, in which the Gauss-Newton method incorporating an nonlinear constant-gain filter for state estimation is used to estimate the system coefficients, has been found to work satisfactorily for moderately nonlinear systems. However, for systems in which the deviations from the nominal conditions (trajectory) are large, or when the nonlinearities dominate the system responses, a steady-state filter may not remain quite adequate. Incorporation of a time-varying filter then becomes essential.

The computational and implementation requirements of a time-varying filter for state estimation within the iterative Gauss-Newton optimization method for parameter estimation, as will be discussed in this paper, are quite complex. Nevertheless, for linear systems, it still provides an optimal state estimator. Because of this reason, the other indirect approach, called filtering- or state-augmentation approach, in which the extended Kalman filter provides estimates of the augmented states (unknown parameters) has been rarely used to estimate the aerodynamic derivatives from linear models.[22]

This paper, however, is mainly concerned with the parameter estimation in nonlinear systems. Even the use of the direct approach, i.e., the Gauss-Newton method with a suitable state estimator, will invariably involve approximations. Optimal filters for general nonlinear systems are practically unrealizeable except for some simple cases. As such, it will be interesting and also worth consideration to evaluate the merits and limitations of the preceding two different approaches in the current context.

Four different estimation algorithms that enable parameter estimation in systems with additive process and measurement noise are critically evaluated from a viewpoint of computation complexity and time, convergence properties, parameter estimates, and their accuracies. Three different model postulates, one linear and two nonlinear, are used for this purpose. An attempt is made to arrive at general conclusions that will enable us to focus further research efforts in a particular direction.

Model Formulation

The dynamic system, whose parameters are to be estimated, is assumed to be described by the following stochastic equations:

$$\dot{x}(t) = f[x(t), u(t), \beta] + F\,w(t) \qquad x(t_0) = x_0 \tag{1}$$

$$y(t) = g[x(t), u(t), \beta] \tag{2}$$

$$z(k) = y(k) + Gv(k) \qquad k = 1, \ldots, N \tag{3}$$

Then n- and m-dimensional system functions f and g are general nonlinear real valued vector functions. These system functions are assumed to have sufficient differentiability to be able to invoke Taylor series expansion. The m-dimensional measurement z is sampled at N discrete time points with a uniform sampling time of Δt.

It is assumed that the process and measurement noise w and v affect the dynamic system linearly. They are assumed to be characterized by a zero-mean white Gaussian noise with an identity spectral density matrix and by a zero-mean, identity covariance Gaussian vector, respectively. The F and G are considered to be time-invariant. Furthermore, the process and measurement noise are assumed to be independent.

It is necessary to estimate β from the discrete measurements of system responses $z(\cdot)$ to given inputs $u(\cdot)$ based on the mixed continuous/discrete system model postulated in Eqs. (1-3). In addition to the unknown system parameters, x_0 are also usually unknown. Furthermore, the measurements of z and u are likely to contain the systematic errors Δz and Δu, respectively, which may also be required to be simultaneously estimated. In a general case, Θ to be estimated is given by

$$\Theta^T = \{\beta^T; x_0^T; \Delta z^T; \Delta u^T\} \tag{4}$$

In many cases, however, it will not be possible to estimate all of the components of x_0, Δu, and Δz, because they may be linearly dependent or at least highly correlated.

Although our primary interest is to estimate β, like other additional parameters x_0, Δu, Δy pertaining to the system variables, F and G may also be generally unknown. Simultaneous estimation of these noise matrices along with Θ is desirable.

Two different approaches to estimate the unknown system parameters in nonlinear systems are possible (Fig. 1). They are 1) filter error methods, in which estimates of the unknown parameters are obtained by minimization of a certain cost function, and 2) filtering approach, in which the unknown parameters are artificially defined as additional state variables and then a nonlinear filter is used to estimate the augmented state vector. The two approaches are briefly presented in the following.

Parameter Estimation by Filter Error Methods

In this direct approach an appropriately defined cost function, usually the statistically formulated likelihood function, is optimized with respect to the unknown parameters using some suitable optimization method. The Gauss-Newton method is generally used for this purpose.[22] Filter is used only for the natural purpose of obtaining the true state variables from the noisy measurements, i.e., for the state estimation. In many cases, for example, when system models are time-invariant and contain weak nonlinearities, it may be adequate to use a steady-state filter. However, if the system response is dominated by nonlinearities or when the deviations from the nominal trajectory are large, it may then be necessary to incorporate a time-varying filter. Both of these possibilities are investigated in this paper.

Time-Varying Filter

The maximum likelihood estimates of the unknown parameters are obtained by minimization of the negative log likelihood function[22]:

$$J(\Theta,R) = \frac{1}{2}\sum_{k=1}^{N}[z(k)-\tilde{y}(k)]^T R^{-1}(k) \times [z(k)-\tilde{y}(k)] + \frac{1}{2}\sum_{k=1}^{N}\ln|R(k)| \tag{5}$$

where $\tilde{y}$ is the filter predicted observation vector. Minimization is to be carried out subject to the postulated system model in Eqs. (1-3). Although the system model is defined in terms of the F and G matrices, the cost function defined above is in terms of R.

Starting from the suitably specified initial values of Θ, the new updated estimates are obtained by using the Gauss-Newton method:

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \tag{6}$$

$$\Delta\Theta = \left\{\sum_k \left[\frac{\partial\tilde{y}}{\partial\Theta}(k)\right]^T R^{-1}(k)\frac{\partial\tilde{y}}{\partial\Theta}(k)\right\}^{-1} \times \left\{\sum_k \left[\frac{\partial\tilde{y}}{\partial\Theta}(k)\right]^T R^{-1}(k)[z(k)-\tilde{y}(k)]\right\} \tag{7}$$

The first term in braces on the right-hand side of Eq. (7) is an approximation of the second gradient $\partial^2 J/\partial\Theta^2$. This approximation helps to reduce the computational costs without significantly affecting the convergence.[22]

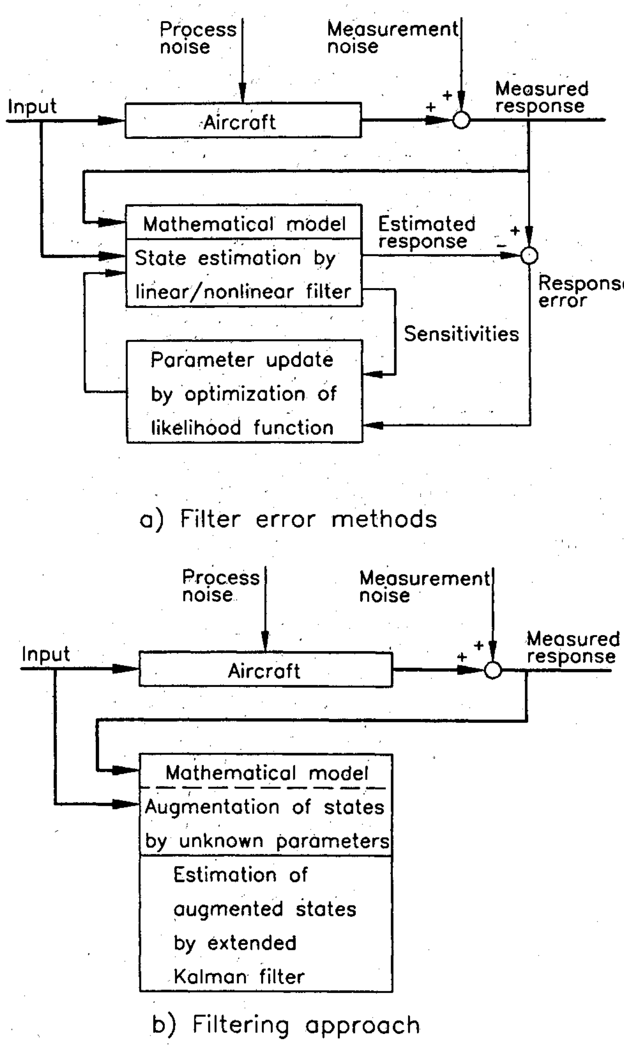

Fig. 1 Parameter-estimation procedures.

The iterative update of Θ using Eqs. (6) and (7) thus requires 1) computation of the predicted observation variables $\tilde{y}(k)$, and 2) computation of the response gradients $(\partial\tilde{y}/\partial\Theta)(k)$ at each time point. Efficient implementation of these computational aspects, together with flexibility to handle conveniently different model structures, is important for parameter estimation in nonlinear systems.

The model predicted response [Eq. (2)] is a function of the parameters being estimated and the state variables $x(t)$. Since the system under investigation contains process noise, it is necessary to incorporate a suitable filter for state estimation. Furthermore, because this paper is mainly concerned with the nonlinear systems, an approximate filter based on a system linearization has been used. The optimal filters for nonlinear systems are practically unrealizable, except for some simple specific cases. The extended Kalman filter (EKF) is one such approximate filter commonly used in nonlinear filtering. It is based on the first-order approximation of the state and measurement equations.[4,5] For the system model defined in Eqs. (1-3), the EKF, which consists of an extrapolation and an update step, can be summarized as

Extrapolation:

$$\tilde{x}(k) = \hat{x}(k-1) + \int_{t_{k-1}}^{t_k} f[x(t), \bar{u}(k), \beta]\,dt \tag{8}$$

$$\tilde{y}(k) = g[\tilde{x}(k), u(k), \beta] \tag{9}$$

$$\tilde{P}(k) \approx \Phi\hat{P}(k-1)\Phi^T + \Delta t F F^T \tag{10}$$

where $\bar{u}(k) = ½[u(k-1) + u(k)]$ and the transition matrix $\Phi = e^{A\Delta t}$ with sampling time $\Delta t = t_k - t_{k-1}$ and A given by

$$A(k) = \left.\frac{\partial f[x(t),u(t),\beta]}{\partial x}\right|_{x=\hat{x}(k-1)} \tag{11}$$

Update:

$$K(k) = \tilde{P}(k)C^T(k)[C(k)\tilde{P}(k)C^T(k) + GG^T]^{-1} \tag{12}$$

$$\hat{x}(k) = \tilde{x}(k) + K(k)[z(k) - \tilde{y}(k)] \tag{13}$$

$$\hat{P}(k) = [I - K(k)C(k)]\tilde{P}(k) = [I - K(k)C(k)]\tilde{P}(k)[I - K(k)C(k)]^T + K(k)GG^T K^T(k) \tag{14}$$

Where C is given by

$$C(k) = \left.\frac{\partial g[x(t),u(t),\beta]}{\partial x}\right|_{x=\tilde{x}(k)} \tag{15}$$

Equation (10) for propagation of the covariance of the state error is an approximation obtained by assuming that Δt is small.[22]

Note here that the actual postulated nonlinear system equations, functions f and g, are used to extrapolate the state estimates ($\tilde{x}$) by numerical integration and for computation of $\tilde{y}$, the predicted system responses. However, the state variable correction in Eq. (13), which depends on K, is based on a first-order system approximation. In case the system dynamics and measurement equations, i.e., functions f and g, are linear, linearizations in Eqs. (11) and (15) are then exact, and the EKF reduces to the Kalman filter, which is an optimal state estimator for linear systems.

In Eq. (14), two forms are provided to compute the updated covariance matrix of the state error $\hat{P}$. Both of the forms are equivalent. However, the longer form is better conditioned for numerical computations and helps to ensure that P always remains positive definite. Hence, this form has been used in the filter implementation, although it increases the computational load.

The time-varying filter further requires the initial state covariance matrix P_0 to be specified. In the case of nonlinear systems, x_0 are estimated along with the other unknown parameters [Eq. (4)]. In such a case, P_0 is zero.[22]

The response gradients $\partial y/\partial \Theta$, as well as the gradients of the system functions f and g in Eqs. (11) and (15), can be evaluated using either the analytical differentiation or finite-difference approximation method. In the case of linear systems, the analytic differentiation method is usually employed.[7,12] However, this requires explicit solution of the sensitivity equations obtained by partial differentiation of the system equations (1) and (2) and also of filter gain given by Eq. (12). For nonlinear systems, the analytical differentiation is tedious and impracticable, requiring model-dependent programming changes for each new nonlinear model form. These practical difficulties are eliminated by finite-difference approximations.[13]

Since the basic interest in this paper, as already pointed out, is estimation in nonlinear systems, the finite-difference approach is used to compute all of the required gradients. For a small perturbation δx_j in each of the n-state variables, the gradients in Eqs. (11) and (15) are approximated using the central difference formulae as

$$A_{ij}(k) \approx \frac{1}{2\,\delta x_j(k)} \{f_i[x(k) + \delta x_j(k)e^j, u(k), \beta] - f_i[x(k) - \delta x_j(k)e^j, u(k), \beta]\}_{x=\hat{x}(k-1)} \tag{16}$$

$$C_{ij}(k) \approx \frac{1}{2\,\delta x_j(k)} \{g_i[x(k) + \delta x_j(k)e^j, u(k), \beta] - g_i[x(k) - \delta x_j(k)e^j, u(k), \beta]\}_{x=\tilde{x}(k)} \tag{17}$$

Likewise, for a small perturbation $\delta\Theta_j$ in each component of Θ, the perturbed response variables $\tilde{y}_{p_j}$ are computed. The corresponding sensitivity coefficient is then approximated by

$$\left[\frac{\partial \tilde{y}}{\partial \Theta}(k)\right]_{ij} \approx \frac{\tilde{y}_{p_i}(k) - \tilde{y}_i(k)}{\delta\Theta_j} \tag{18}$$

The perturbed responses $y_p(k)$ are obtained from the perturbed system equations.

To avoid notational complexity, the perturbed equations for time-varying filter used in the gradient computations are not presented in this paper. It would suffice to mention here that these perturbed equations have a structure similar to the system filter [Eqs. (8–15)]. It is easy to see from Eq. (18) that it requires propagation of the perturbed state variables $x_p(k)$ and of perturbed error-covariance matrix $P_p(k)$ for each element of Θ. However, detailed equations, included for the simplified case of steady-state filter, are discussed in the next section.

All of the quantities required in Eq. (7) to compute iteratively the parameter improvement $\Delta\Theta$ are now defined. The residuals $[z(k) - \tilde{y}(k)]$ are readily available from Eq. (13) in the filter implementation. The covariance matrix of residuals $R(k)$ can be obtained without any additional computations by making use of the relation

$$R(k) = C(k)\tilde{P}(k)C^T(k) + GG^T \tag{19}$$

where the right-hand side is already evaluated in Eq. (12).

For filter error methods using a steady-state filter, the filter implementation can be done in terms of matrix R, as in the next section.[6,7,12] This is found to be convenient because the cost function in Eq. (5) is defined in terms of R and also the steady-state R can be estimated using a closed-form solution.[22] It is also possible to implement the steady-state filter in terms of the G matrix. From the explicitly estimated steady-state R matrix, the measurement noise covariance matrix GG^T can be derived with only minor additional computations.[11]

For a time-varying filter, the maximum likelihood estimate of $R(k)$ can be obtained by setting the partial derivative of Eq. (5) with respect to $R(k)$ to zero. The yields

$$\hat{R}(k) = [z(k) - \tilde{y}(k)][z(k) - \tilde{y}(k)]^T \tag{20}$$

The time-varying filter implementation, however, requires knowledge of G matrix. Although Eq. (20) provides the maximum likelihood estimate of R at each time point, it appears that a simple procedure to obtain updated G estimates from the updated $\hat{R}(k)$, similar to the one in the steady-state case,[11] is not feasible for a time-varying filter.

In view of the preceding brief discussion, estimation of GG^T using a time-varying filter is not considered in this paper. However, this limitation is not considered to be a serious drawback, since reasonable information about the G matrix can be obtained from the laboratory calibrations of the various measurement sensors used. In addition, estimates of the system parameters are relatively insensitive to the covariance matrix estimates.[7] For the reason last mentioned, F can also be kept constant using estimates of F obtained from other flight records.

Steady-State Filter

In many applications, particularly when the system under investigation is time-invariant and the deviations from the nominal trajectory are small, it is often adequate to use a steady-state filter for state estimation. This simplification results in significant reduction of computational burden. The cost function defined in Eq. (5) reduces in such a case to

$$J(\Theta, R) = \frac{1}{2}\sum_{k=1}^{N} [z(k) - \tilde{y}(k)]^T R^{-1} [z(k) - \tilde{y}(k)] + \frac{N}{2}\ell n|R| \tag{21}$$

where R is the steady-state covariance matrix of the residuals.

The parameter improvement vector $\Delta\Theta$ in this case is obtained from Eq. (7) by replacing the time-varying $R(k)$ with a steady-state R as

$$\Delta\Theta = \left\{\sum_k \left[\frac{\partial \tilde{y}}{\partial \Theta}(k)\right]^T R^{-1} \frac{\partial \tilde{y}}{\partial \Theta}(k)\right\}^{-1} \times \left\{\sum_k \left[\frac{\partial \tilde{y}}{\partial \Theta}(k)\right]^T R^{-1} [z(k) - \tilde{y}(k)]\right\}^{-1} \tag{22}$$

As already pointed out, computation of the filter predicted observation variables and its gradients are the two important aspects of the estimation algorithm. In the case of nonlinear systems, x_0 are updated iteratively in the parameter-estimation loop. Linearization of the system equations in each iteration about this convenient point x_0 can be used to compute the constant-gain matrix. Such a nonlinear constant-gain filter can be represented as

$$\tilde{x}(k) = \hat{x}(k-1) + \int_{t_{k-1}}^{t_k} f[x(t), \bar{u}(k), \beta]\,dt \tag{23}$$

$$\tilde{y}(k) = g[\tilde{x}(k), u(k), \beta] \tag{24}$$

$$\hat{x}(k) = \tilde{x}(k) + K[z(k) - \tilde{y}(k)] \tag{25}$$

The steady-state K is a function of the steady-state R, the steady-state P, and C of the linearized system. It is given by

$$K = PC^TR^{-1} \tag{26}$$

where

$$C = \left[\frac{\partial g[x(t),u(t),\beta]}{\partial x}\right]_{t=t_0} \tag{27}$$

A steady-state form of the Riccati equation has been used here to obtain P. Furthermore, continuous-time Riccati equation, which can be solved efficiently is preferred to the discrete-time equation. The first-order approximation of this equation is obtained as[7]

$$AP + PA^T - \frac{1}{\Delta t}PC^TR^{-1}CP + FF^T = 0 \tag{28}$$

where

$$A = \left\{\frac{\partial f[x(t),u(t),\beta]}{\partial x}\right\}_{x=x_0} \tag{29}$$

represents the state matrix of the linearized system. Eigenvector decomposition method has been used to solve Eq. (28).[23,24] Matrix P obtained in the preceding equations does not depend on time. Equation (26), in turn, provides a constant-gain matrix corresponding to the system linearization about x_0, which are updated in each iteration.

As before, all of the required gradients are approximated using the finite-difference approximation. The system matrices A and C are obtained by central difference formulae from Eqs. (16) and (17) evaluated at $t = t_0$, only once in each iteration.[21] The sensitivity coefficients $\partial y/\partial \Theta$ are obtained from Eq. (18).

The perturbed response variables y_p required in Eq. (18) to compute the sensitivity coefficients are obtained from the perturbed-system equations. For each element of Θ, the state and observation variables can be propagated according to[21]

$$\tilde{x}_p(k) = \hat{x}_p(k-1) + \int_{t_{k-1}}^{t_k} f[x_p(t),\bar{u}(k),\beta + \delta\beta]\,\mathrm{d}t \tag{30}$$

$$\tilde{y}_p(k) = g[\tilde{x}_p(k),u(k),\beta + \delta\beta] \tag{31}$$

$$\hat{x}_p(k) = \tilde{x}_p(k) + K_p[z(k) - \tilde{y}_p(k)] \tag{32}$$

Computation of the predicted perturbed-state variables $\tilde{x}_p$ [Eq. (30)] by numerical integration and of perturbed output variables $\tilde{y}_p$ [Eq. (31)] is straightforward. However, computation of the updated state variables [Eq. (32)] additionally requires perturbed-gain matrix K_p, which is different for each parameter perturbation:

$$K_p = P_pC_p^TR^{-1} \tag{33}$$

where the covariance matrix of the state error P_p for a particular perturbed parameter is obtained by solving the Riccati equation [Eq. (28)], with system matrices computed for the corresponding perturbations. The elements of A_p and C_p corresponding to each parameter perturbation $\delta\beta$ are once again approximated by central difference formulae given in Eqs. (16) and (17) evaluated at $\beta + \delta\beta$ and $x = x_0$ once in each iteration.

The extension of finite-difference approximation to filter error algorithm thus involves not only numerical integration of the perturbed-state equations but also computation of the perturbed-gain matrices for each element of Θ. The perturbed output variables y_p computed from Eq. (31) using perturbed states x_p and perturbed gain K_p will automatically account for the respective gradients. Thus, in comparison to the previous formulation for linear systems in Refs. 7 and 12, the solution to a set of Lyapunov equations for the gradient of P has been replaced in the present approach by solutions to the perturbed Riccati equations. Increase in the computational load due to this change will not be significantly high.

The two-step optimization procedure yields iteratively Θ and R. This has been done with a view to eliminate the convergence difficulties,[7] which are generally encountered in explicit estimation of covariance matrix GG^T. Hence, the measurement noise covariance matrix in the present case can only be indirectly obtained from the relation

$$R = GG^T + CPC^T \tag{34}$$

For physically meaningful results, it is necessary to ensure that GG^T, as will be obtained indirectly from the preceding equation, is positive semidefinite. It has been shown in Ref. 7 that, for linear systems, the preceding conditions are well-approximated by constraining the diagonal elements of the matrix KC to be less than unity. Minimization of the cost function [Eq. (21)] subject to these nonlinear inequality constraints leads to a nonlinear programming problem, which is solved by a quadratic programming method.

In the present case, a similar approach has been adopted with a further simplification that the elements of KC are constrained to be less than unity, where the observation matrix C obtained by first-order system approximation has been used. The computational details of constrained optimization are similar to those found in Refs. 7 and 12, and, hence, are not discussed further in this paper.

In the derivation of estimation algorithm, it has been hitherto assumed that the covariance matrix of innovations R is known. In practice, however, it may not be known accurately. The maximum likelihood estimate of R can be obtained by equating the gradient of the cost function [Eq. (21)] with respect to R to zero. Since the innovations are functions of K, the exact equation for R is complex and computationally tedious. However, as shown in Ref. 7, the asymptotic approximation to R can be obtained as

$$R = \frac{1}{N}\sum_{k=1}^{N}[z(k) - \tilde{y}(k)][z(k) - \tilde{y}(k)]^T \tag{35}$$

The two steps to compute $\Delta\Theta$ and R [Eqs. (22) and (35)] are carried out independently. Hence, they do not account for the influence of each on estimates of the other. This often yields strongly correlated estimates of F and R, which affects the convergence. To account for this correlation effect, the following heuristically suggested approximate procedure employed in Ref. 7 has been used here to compensate for the F matrix, whenever R is revised:

$$F_{ij}^{\text{new}} = F_{ij}^{\text{old}}\left(\frac{\sum_k C_{ki}^2 r_k^{\text{old}}\sqrt{r_k^{\text{old}}/r_k^{\text{new}}}}{\sum_k C_{ki}^2 r_k^{\text{old}}}\right) \tag{36}$$

where r_k is the kth diagonal element of inverse of R and the superscripts old and new denote the previous and revised estimates, respectively.

Although F is an $(n \times n)$ matrix [Eq. (1)], it is common practice in estimation to treat it as a diagonal matrix. This simplification not only helps to reduce the additional computational burden but also to avoid any identification problems.

In the case of filter error methods using either a time-varying or a steady-state filter, the inverse of the second gradient of the cost function with respect to the unknown parameter vector provides an approximation to the parameter error covariance matrix. This enables to compute readily the standard deviations of the estimates and the correlation coefficients between them.[12,22]

Parameter Estimation by Filtering Approach

In this indirect approach the parameter estimation problem is transformed into a state estimation problem by artificially defining the unknown system parameters as additional state variables. Considering the constant system parameters Θ as output of an auxiliary dynamic system,

$$\dot{\Theta} = 0 \tag{37}$$

and by defining an augmented state vector

$$x_a^T = \{x^T, \Theta^T\} \tag{38}$$

the extended system can be represented as

$$\dot{x}_a(t) = f_a[x_a(t),u(t)] + F_a w_a(t) \tag{39}$$

$$= \begin{bmatrix} f[x(t),u(t),\beta] \\ 0 \end{bmatrix} + \begin{bmatrix} F|0 \\ 0|0 \end{bmatrix} \begin{bmatrix} w(t) \\ 0 \end{bmatrix}$$

$$y(t) = g_a[x_a(t),u(t)] \tag{40}$$

$$z(k) = y(k) + Gv(k) \tag{41}$$

For the preceding augmented system the EKF is obtained from Eqs. (8-17) with x, A, P, and C replaced, respectively, by x_a, A_a, P_a, and C_a. Although the matrices A_a and C_a for the linearized augmented system are now of larger dimensions, many of the elements of A_a are zero.[25] Furthermore, like all of the state estimation techniques, the combined state and parameter estimation using the EKF is a single-pass procedure. The filtering approach leads to a nonlinear augmented system even for systems, i.e., functions f and g, which are originally linear.

A priori specification of the covariance matrices of the process and measurement noise is necessary to use the EKF as a parameter estimator. The problem of estimating the unknown noise statistics for a known system dynamics, often referred to in the literature as "adaptive filtering," is not addressed in this paper.[4,5] Likewise, estimation of x_0, which appear only indirectly in the postulated state equations, is also not considered. The state variables are treated as random variables with a mean x_0 and covariance $P(0)$, both of which are to be specified a priori.

It is required to specify in the present case the initial covariance matrix $P_a(0)$ for the augmented state vector x_a. The initial $P(0)$ corresponding to the auxiliary state variables (unknown parameters) should indicate the confidence in the starting values of Θ. In the absence of any a priori knowledge, it is common to assume a fairly high value for $P_a(0)$.

The EKF is only a first-order minimum variance filter. The higher-order terms neglected in propagation of the error covariances can possibly lead to biased estimates.[18,26] Various techniques, such as iterated extended Kalman filter or second- and higher-order filters, are often used in many applications to reduce the estimation errors.[4,5] However, such modifications are not investigated for the basic study aimed at in this paper.

The covariance matrix P_a provides the information about the accuracy of the states (unknown parameters). Standard deviations of the parameter estimates can be readily obtained as square root of the corresponding diagonal element of P_a.

Summary of Various Estimation Algorithms

To facilitate easy reference, the three estimation algorithms discussed above are summarized in Table 1. In addition, an algorithm developed specifically for linear systems only and used in the examples analyzed in this paper has also been included. Detailed description of this algorithm is found in Ref. 12.

The abbreviations L and NL are used in Table 1 to denote the linear and nonlinear system models, respectively, and indicate the scope of the various algorithms.

In the program packages KALML7 and NLMLKL, R has been constrained to be a diagonal matrix. This constraint helps to reduce the computations in the second gradient of the cost function. In the case of output error method, it is also adequate and aids in improving the convergence.[27]

The program package MLEKF, using a time-varying filter, provides an option to estimate F. However, to facilitate comparison of the results with those obtained using the estimation program XAEKF in which F is required to be specified a priori, this option has not been used.

Examples

The first example pertains to estimation of the aircraft lateral-directional derivatives. Simulated aircraft responses and a linear model are used, both of which enable critical evaluation of the algorithms, and also a comparison of estimates with the nominal parameter values. The other example pertains to extraction of the aircraft longitudinal derivatives from flight-test data. Two nonlinear models in different axes systems with different degrees of nonlinearities are used for this purpose.

Lateral-Directional Motion – Simulation with Process Noise

To evaluate performance of the estimation algorithms on data with appreciable level of turbulence, typical aircraft responses pertaining to lateral-directional motion are generated through simulation. Nominal values of the aerodynamic derivatives used correspond to those obtained by parameter estimation from flight data recorded during the tests in a steady atmosphere with the research aircraft de Havilland DHC-2.[28] Standard equations of aircraft motion[29] incorporating additional state and measurement noise are used to generate the data. To provide realistic control inputs, the rudder and aileron excitations actually applied in a particular flight test are used. For the purpose of simulation, independent process and measurement noise vectors are generated using a pseudorandom noise generator. State noise matrix is assumed to be diagonal. A total of 16 s of data with a sampling time of 0.05 s has been generated. These typical noisy simulated aircraft responses are used as measured data for the purpose of estimation.

The following model pertaining to the lateral-directional motion of aircraft has been used to estimate the dimensional derivatives.[29,30]

State equations:

$$\dot{p} = L_p p + L_r r + L\delta_a \delta a + L\delta_r \delta r + L_v v + b_{\dot{x}_p}$$

$$\dot{r} = N_p p + N_r r + N\delta_a \delta a + N\delta_r \delta r + N_v v + b_{\dot{x}_r} \tag{42}$$

Observation equations:

$$\dot{p}_m = L_p p + L_r r + L\delta_a \delta a + L\delta_r \delta r + L_v v + b_{\dot{x}_p}$$

$$\dot{r}_m = N_p p + N_r r + N\delta_a \delta a + N\delta_r \delta r + N_v v + b_{\dot{x}_r}$$

$$a_{y_m} = Y_p p + Y_r r + Y\delta_a \delta a + Y\delta_r \delta r + Y_v v + b_{y_{ay}}$$

$$p_m = p + b_{y_p}$$

$$r_m = r + b_{y_r} \tag{43}$$

Table 1 Four estimation algorithms

Estimation algorithm	Criterion	Parameter update	State estimation	Option to estimate F	Option to estimate R/G	System models
KALML7 (Ref. 12)	Maximum likelihood	Gauss-Newton	Steady-state filter	Yes	Yes	L
NLMLKL (Ref. 21)	Maximum likelihood	Gauss-Newton	Steady-state filter	Yes	Yes	L/NL
MLEKF (Ref. 25)	Maximum likelihood	Gauss-Newton	Time-varying filter (EKF)	Yes	No	L/NL
XAEKF (Ref. 25)	Nonlinear filtering	Time-varying filter EKF	EKF	No	No	L/NL

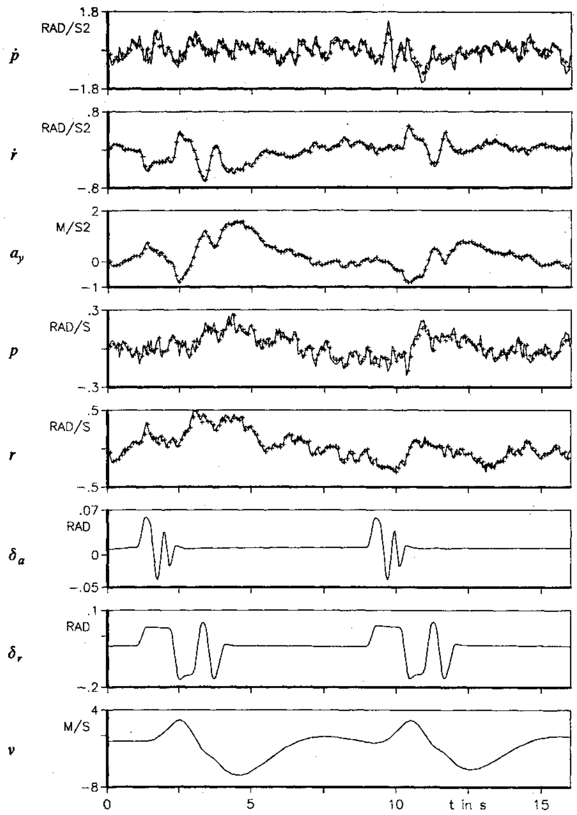

Fig. 2 Comparison of measured (—) and estimated (+ + + +) responses pertaining to lateral-directional aircraft motion.

The Θ for the preceding model consists of the dimensional derivatives and the bias parameters. It is given by

$$\Theta^T = [L_p\ L_r\ L_{\delta_a}\ L_{\delta_r}\ L_v\ N_p\ N_r\ N_{\delta_a}\ N_{\delta_r}\ N_v$$
$$Y_p\ Y_r\ Y_{\delta_a}\ Y_{\delta_r}\ Y_v\ b_{\dot{x}_p}\ b_{\dot{x}_r}\ b_{\dot{y}_p}\ b_{\dot{y}_r}\ b_{y_{ay}}\ b_{y_p}\ b_{y_r}] \qquad (44)$$

For linear systems, the lumped bias parameters $b_{x(\cdot)}$ and $b_{y(\cdot)}$ provide a convenient means to appropriately account for the initial conditions x_0 and the zero shifts Δu, Δy [Eq. (4)]. Furthermore, the initial conditions to be used in the computational algorithm reduce to zero.[27] This helps to set the a priori value of the initial state covariance matrix $P(0)$ to zero required in the time-varying filters.

To evaluate the effect of initial parameter values on the convergence, different sets of starting values have been tried.[25] The values that are quite far off from the nominal are chosen here for the presentation. The results of parameter estimation from the four estimation algorithms, KALML7, NLMLKL, MLEKF, and XAEKF, are provided in Table 2. The matrices F and G have been specified a priori and held constant. All of the algorithms yielded estimated responses that match well with the measured responses. There was hardly any qualitative difference in the time plots provided by different algorithms. Hence, only a single typical plot shown in Fig. 2 has been provided. In addition, typical convergence plots of some of the augmented states (unknown parameters) obtained from the single-pass algorithm XAEKF are found in Fig. 3.

The study of Table 2 and Fig. 3 leads to the following general observations:

1) All of the four algorithms converge to the respective minimum. In general, estimates of all the derivatives provided

Table 2 Dimensional derivatives pertaining to lateral-directional aircraft motion

			Estimates[a]			
Parameter	Nominal value	Starting value	Algorithm KALML7	Algorithm NLMLKL	Algorithm MLEKF	Algorithm XAEKF
L_p	−5.820	−10.0	−5.722(6.2)[b]	−5.721(6.6)	−5.684(6.2)	−5.867(6.1)
L_r	1.782	4.00	1.724(8.8)	1.721(9.1)	1.747(8.1)	1.809(7.6)
N_p	−0.665	−1.50	−0.620(8.6)	−0.620(8.8)	−0.654(6.3)	−0.673(8.0)
N_r	−0.712	−1.50	−0.722(3.3)	−0.723(3.3)	−0.717(2.9)	−0.718(2.9)
L_{δ_a}	−16.434	−6.00	−14.929(11)	−14.927(11)	−15.726(9.0)	−15.429(9.2)
L_{δ_r}	0.434	0.80	0.202(214)	0.200(216)	0.394(92)	0.347(99)
L_v	−0.097	−0.30	−0.088(13)	−0.088(13)	−0.092(10)	−0.094(10)
N_{δ_a}	−0.428	−0.20	−0.426(59)	−0.0424	−0.429(50)	−0.411(51)
N_{δ_r}	−2.824	−1.40	−2.828(2.3)	−2.828(2.3)	−2.823(1.8)	−2.836(1.9)
N_v	0.0084	0.08	0.0093(18)	0.0093(18)	0.0086(17)	−0.0084(17)
Y_p	−0.278	−0.50	−0.297(27)	−0.297(27)	−0.289(27)	−0.307(26)
Y_r	1.410	3.00	1.415(2.4)	1.415(2.5)	1.421(2.3)	1.436(2.2)
Y_{δ_a}	−0.447	−0.25	−0.514(73)	−0.517(72)	−0.474(70)	−0.477(66)
Y_{δ_r}	2.657	5.50	2.688(3.7)	2.688(3.7)	2.682(3.0)	2.676(3.0)
Y_v	−0.180	−0.08	−0.180(1.5)	−0.180(1.5)	−0.180(1.2)	−0.180(1.2)
Iterations			9	9	7	——
CPU time, s			22	45	305	31

[a]Estimates were obtained by accounting for both process and measurement noise.
[b]Values in parentheses indicate standard deviations in %.

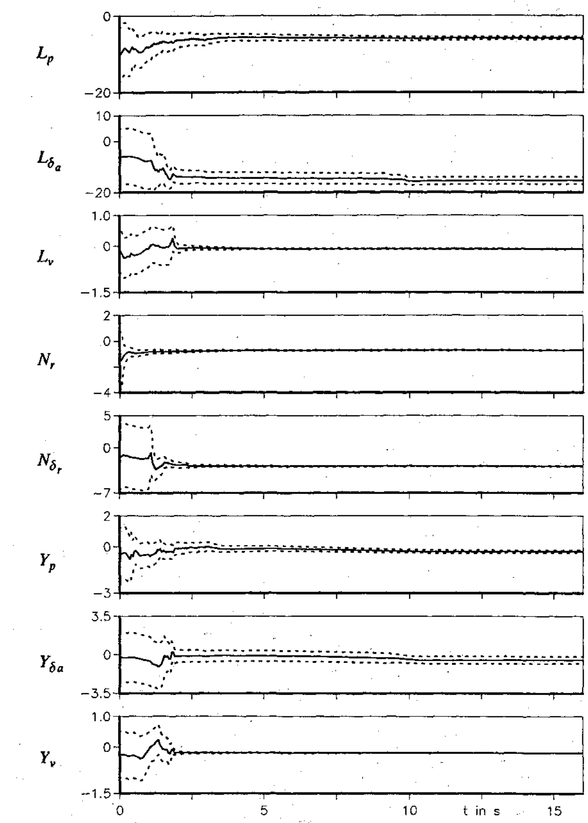

Fig. 3 Convergence of the dimensional derivatives estimated using the extended Kalman filter: —, estimate; ---, estimate ± standard deviation.

by the various algorithms match reasonably well with the expected nominal values.

2) The estimates provided by the algorithm NLMLKL using an approximate filter with constant gain are identical to those obtained from the estimation program for linear system only, KALML7. This is to be expected, since, for linear models the linearization of system functions f and g is exact. The approximate filter reduces to the optimal Kalman filter. Thus, for linear systems the two algorithms are equivalent. The algorithm NLMLKL needs more computational time due to the use of finite-difference approximation in gradient computations and also of Runge-Kutta numerical integration.

3) In general, it has been found that the estimates provided by the algorithms MLEKF and XAEKF, using a time-varying filter, are closer to the nominal values than those obtained from KALML7 and NLMLKL (with a constant gain filter). This is true not only for the aerodynamic derivatives, but also for the control derivatives such as L_{δ_r}, Y_{δ_a}. The preceding differences are due to the fact that in the algorithms KALML7 and NLMLKL the residual covariance matrix R is constrained to be diagonal. On the other hand, the time-varying filter in MLEKF and XAEKF automatically leads to a full matrix.

4) Although the algorithm MLEKF using a time-varying filter requires considerably more computational time, no numerical difficulties are encountered.

5) The filtering approach to parameter estimation, algorithm XAEKF, yields estimates that tend to agree well with those obtained from MLEKF. This is attributed to the use of a time-varying filter in both the algorithms. In the filtering approach, algorithm XAEKF, most of the augmented state variables (aerodynamic derivatives) are found to converge within the first 5 s (Fig. 3), which corresponds to the first excitation of the system. Only some of the control derivatives Y_{δ_a}, L_{δ_a} are found to change a little with the additional information due to the second excitation.

6) For the one-pass algorithm XAEKF, irrespective of the starting values, the computational time depends only on the number of unknown parameters, state variables, and data points. On the other hand, for the algorithms KALML7, NLMLKL, and MLEKF apart from the previously mentioned dependence, the convergence also depends on the starting parameter values. In general, the iterative algorithms require more iterations, when the starting values are far from the optimum.

It is found that the output error method is quite inadequate for the analysis of this data with appreciable level of turbulence. Comparison of the results by accounting for and neglecting the process noise for this example are found in Ref. 12.

For the preceding example, as well as for the examples presented below, a fourth-order Runge-Kutta method is used to integrate the system and perturbed-state equations. The response gradients in Eq. (18) are computed using a step size $\delta\Theta$ of $(10^{-6}\ \Theta)$. The first-order approximations of system matrices A and C in Eqs. (16), (17), (27), and (29) are obtained using a perturbation δx of $(10^{-4}\ x)$.

Estimation of Longitudinal Derivatives

The second and third example pertain to estimation of the longitudinal derivatives of a research aircraft HFB-320. Flight tests were carried out to excite the longitudinal motion of the aircraft.[17] Although a linear model can be formulated to extract such information, this paper considers mainly nonlinear models, which are in general found to provide improved estimation results.[16,17,31]

It is possible to formulate the system (aircraft motion) equations using different coordinate systems. Proper choice of axis system is of importance for parameter estimation, since it leads to different degrees of nonlinearities in the postulated models.[2] The simplest of the nonlinear model, containing only the multiplicative and trigonometric nonlinearities, is obtained by defining the normalized aerodynamic forces X and Z and normalized pitching moment M as functions of variables in the body axes (u, w). Experience indicates that such nonlinear models in terms of the dimensional derivatives $(X_u, Z_w, M_q, \ldots)$[29,30] pose no specific difficulties. Some results obtained from such models using the algorithm NLMLKL are found in Ref. 21. In many cases, however, it is of interest to obtain the nondimensional aircraft derivatives.[2,29] Furthermore, the use of state and observation equations formulated directly in terms of nondimensional coefficients has been found to provide significantly improved estimation results.[16,17] Hence, two such models analyzed below are considered here.

Defining C_X, C_Z, and C_m as functions of variables in the body axes $(u, w, \text{etc.})$, the following system model is obtained:

State equations:

$$\dot{u} = \frac{\bar{q}S}{m}\left(C_{X_0} + C_{X_u}\frac{u}{V_0} + C_{X_w}\frac{w}{V_0}\right) - qw - g\sin\theta + \frac{F_e}{m}\cos\sigma_T \qquad u(0) = u_0 \tag{45a}$$

$$\dot{w} = \frac{\bar{q}S}{m}\left(C_{Z_0} + C_{Z_u}\frac{u}{V_0} + C_{Z_w}\frac{w}{V_0}\right) + qu + g\cos\theta - \frac{F_e}{m}\sin\sigma_T \qquad w(0) = w_0 \tag{45b}$$

$$\dot{\theta} = q \qquad \theta(0) = \theta_0 \tag{45c}$$

$$\dot{q} = \frac{\bar{q}S\bar{c}}{I_y}\left(C_{m_0} + C_{m_u}\frac{u}{V_0} + C_{m_w}\frac{w}{V_0} + C_{m_q}\frac{\bar{c}}{2V_0}q + C_{m_{\delta e}}\delta_e\right) \qquad q(0) = q_0 \tag{45d}$$

Observation equations:

$$V_m = \sqrt{u^2 + w^2} \tag{46a}$$

$$\alpha_m = \tan^{-1}\left(\frac{w}{u}\right) \tag{46b}$$

$$\theta_m = \theta \tag{46c}$$

$$q_m = q \tag{46d}$$

$$\dot{q}_m = \frac{\bar{q}S\bar{c}}{I_y}\left(C_{m_0} + C_{m_u}\frac{u}{V_0} + C_{m_w}\frac{w}{V_0} + C_{m_q}\frac{\bar{c}}{2V_0}q + C_{m_{\delta e}}\delta_e\right) \tag{46e}$$

$$a_{x_m} = \frac{\bar{q}S}{m}\left(C_{X_0} + C_{X_u}\frac{u}{V_0} + C_{X_w}\frac{w}{V_0}\right) + \frac{F_e}{m}\cos\sigma_T \tag{46f}$$

$$a_{z_m} = \frac{\bar{q}S}{m}\left(C_{Z_0} + C_{Z_u}\frac{u}{V_0} + C_{Z_w}\frac{w}{V_0}\right) - \frac{F_e}{m}\sin\sigma_T \tag{46g}$$

The preceding system equations in terms of variables in the body axis (u,w) contain not only the common trigonometric and multiplicative nonlinearities, but also those introduced in the measurement equations due to the use of directly measured variables V and α. In addition, the variable dynamic pressure $\bar{q}(=\frac{1}{2}\rho V^2)$, which multiplies all of the aerodynamic derivatives, introduce additional nonlinearities.

The unknown Θ consisting of the nondimensional derivatives and initial conditions is given by

$$\Theta^T = [C_{X_0}, C_{X_u}, C_{X_w}, C_{Z_0}, C_{Z_u}, C_{Z_w}, C_{m_0}, C_{m_u}, C_{m_w}, C_{m_q}, C_{m_{\delta e}}, u_0, w_0, \theta_0, q_0] \tag{47}$$

A particular record, that appears to contain atmospheric turbulence is analyzed here.[12,21] A record length of 60 s with a sampling time of 0.1 s is used.

Since the postulated model in Eqs. (45) and (46) is nonlinear, obviously the program KALML7 applicable to linear systems only cannot be used. The results of parameter estimation accounting for both process and measurement noise using the three algorithms NLMLKL, MLEKF, and XAEKF are summarized in Table 3. For this case, i.e., when the starting values were far from the optimum, the output error method did not converge even after several iterations.[25] However, starting from parameter values closer to the optimum, the output error method converged, yielding plots shown in Fig. 4. The differences in the measured and estimated responses are discernible. Furthermore, the estimation residuals were found not to be white noise.

On the other hand, the agreement between the estimated and measured responses in Fig. 5 is very good and clearly brings out the improvements obtained by accounting for the process noise. As in the last example, since no qualitative difference are found in the time history plots obtained from different algorithms, only a single plot has been provided. Furthermore, it has been found that the spectra of residuals

Table 3 Estimation of aircraft longitudinal derivatives from flight-test data: nonlinear model with nondimensional derivatives (body axis)

		ML estimates obtained by accounting for			
		Measurement noise only	Both process and measurement noise		
Parameter	Starting value	Algorithm NLMLKL	Algorithm NLMLKL	Algorithm MLEKF	Algorithm XAEKF
C_{X_u}	0.125		0.0141(16)[a]	0.0140(13)	0.0146(12)
C_{X_w}	1.564		0.6161(1.0)	0.6134(0.9)	0.6111(0.9)
C_{X_0}	−0.348		−0.1160(2.1)	−0.1157(1.8)	−0.1161(1.8)
C_{Z_u}	0.353		0.2851(5.5)	0.2865(4.8)	0.2832(4.8)
C_{Z_w}	−8.091		−4.1902(1.1)	−4.1604(1.0)	−4.1445(1.0)
C_{Z_0}	−0.437		−0.3607(4.9)	−0.3654(4.2)	−0.03636(4.2)
C_{m_u}	0.103		0.1491(23)	0.1070(2.8)	0.1072(2.9)
C_{m_w}	−1.808		0.9252(1.1)	−0.9502(1.0)	−0.9519(1.1)
C_{m_q}	43.440		−33.283(2.3)	−36.913(2.0)	−37.316(1.9)
$C_{m_{\delta e}}$	−3.457		−1.5123(1.1)	−1.5909(1.1)	−1.6018(1.1)
C_{m_0}	0.099		0.1491(2.3)	0.0069(44)	0.0069(47)
u_0	100.0		105.379(0.1)	105.372(0.04)	——
w_0	21.89		11.8254(0.7)	11.8708(0.5)	——
θ_0	0.929		0.1049(0.3)	0.1048(0.2)	——
q_0	−0.096		−0.00332(17)	−0.00349(5.8)	——
Iterations		No convergence even after several iterations	5	4	——
CPU time, s			58	550	32

[a]Values in parentheses indicate standard deviations in %.

are flat, indicating that they are essentially white. Figure 6 provides the convergence plots of some of the unknown parameters obtained using the algorithm XAEKF.

The study of Table 3 and Fig. 6 leads to the following observation for this example:

1) All the filter algorithms converge to the respective minimum. On the other hand, convergence difficulties have been encountered in the output error method, when the starting values are far from the optimum. Thus, the filter algorithms, which inherently provide stabilization in the computations through a feedback, appear to perform better.

2) The estimates of aerodynamic derivatives obtained from the algorithms with time-varying filters, MLEKF and XAEKF, are close to each other. Moreover, they agree well with those provided by the algorithm NLMLKL using a constant gain. Some differences are observed in the estimates of C_{m_q}, $C_{m_{\delta e}}$, C_{m_0}. This is attributed mainly to the various approximations involved in the use of a constant-gain filter and to the use of a diagonal R.

3) The estimates of initial conditions u_0, w_0, θ_0, and q_0, obtained from the algorithms NLMLKL and MLEKF, agree reasonably well.

4) Although the algorithm MLEKF with time-varying filter requires considerably more computational time, no numerical difficulties are encountered.

5) The estimates of all the augmented states (unknown parameters), provided by the algorithm XAEKF, are found to converge quite fast, essentially in the first 20 s. (See Fig. 6).

The aerodynamic derivatives in the wind axis system (lift and drag derivatives) can now be obtained through standard axes transformation from the body-axis nondimensional derivatives $C_{X(\cdot)}$ and $C_{Z(\cdot)}$ estimated using Eqs. (45) and (46). It is also possible to estimate the lift and drag derivatives directly by incorporating such transformations in the postulated system model. Such a model in terms of nondimensional lift, drag, and pitching moment coefficients (C_L, D_D, C_m) as

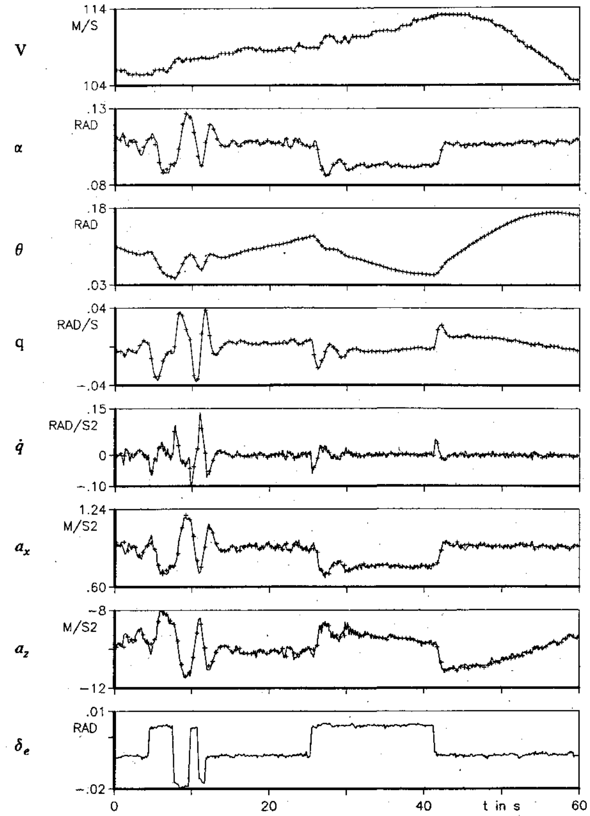

Fig. 5 Comparison of flight measured (—) and estimated (+ + + +) responses by accounting for both process and measurement noise and using a nonlinear model.

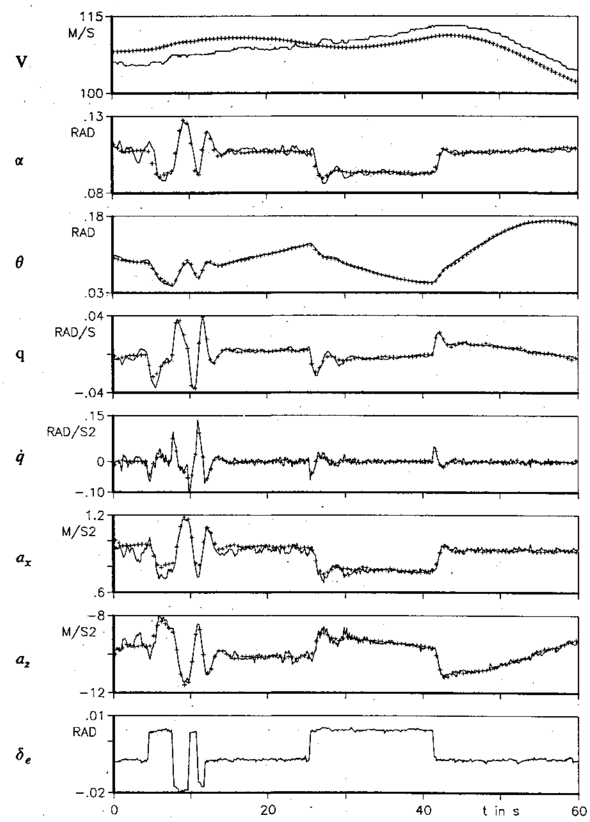

Fig. 4 Comparison of flight measured (—) and estimated (+ + + +) responses obtained using output error method and a nonlinear model.

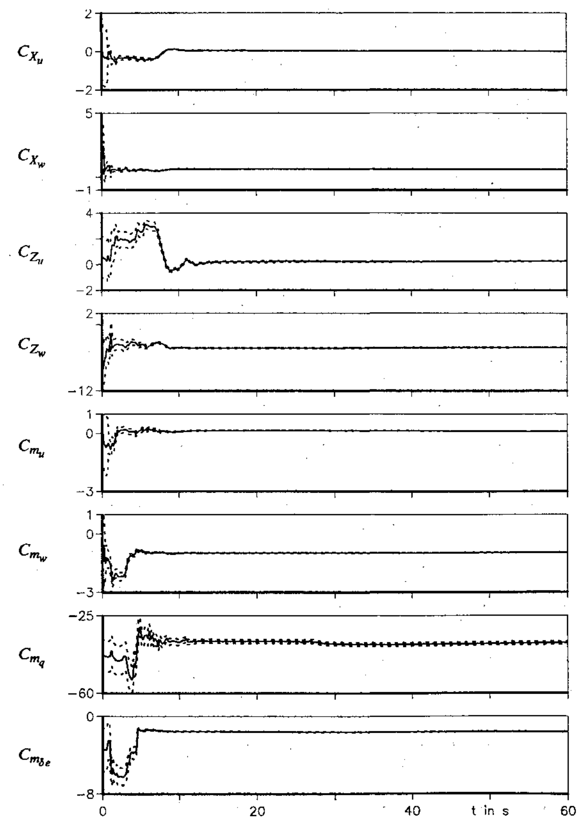

Fig. 6 Convergence of the nondimensional derivatives (body axis) estimated using the extended Kalman filter: —, estimate; ---, estimate ± standard deviation.

functions of variables in the wind axis (V, α, etc.) is given by

State equations:

$$\dot{V} = -\frac{\bar{q}S}{m}\left(C_{D_0} + C_{D_V}\frac{V}{V_0} + C_{D_\alpha}\alpha\right) + g\sin(\alpha - \theta) + \frac{F_e}{m}\cos(\alpha + \sigma_T), \qquad V(0) = V_0 \tag{48a}$$

$$\dot{\alpha} = -\frac{\bar{q}S}{mV}\left(C_{L_0} + C_{L_V}\frac{V}{V_0} + C_{L_\alpha}\alpha\right) + q + \frac{g}{V}\cos(\alpha - \theta) - \frac{F_e}{mV}\sin(\alpha + \sigma_T), \qquad \alpha(0) = \alpha_0 \tag{48b}$$

$$\dot{\theta} = q, \qquad \theta(0) = \theta_0 \tag{48c}$$

$$\dot{q} = \frac{\bar{q}S\bar{c}}{I_y}\left(C_{m_0} + C_{m_V}\frac{V}{V_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{\bar{c}}{2V_0}q + C_{m_{\delta e}}\delta_e\right), \qquad q(0) = q_0 \tag{48d}$$

Observation equations:

$$V_m = V \tag{49a}$$

$$\alpha_m = \alpha \tag{49b}$$

$$\theta_m = \theta \tag{49c}$$

$$q_m = q \tag{49d}$$

$$\dot{q}_m = \frac{\bar{q}S\bar{c}}{I_y}\left(C_{m_0} + C_{m_V}\frac{V}{V_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{\bar{c}}{2V_0}q + C_{m_{\delta e}}\delta_e\right) \tag{49e}$$

$$a_{x_m} = \frac{\bar{q}S}{m}C_X + \frac{F_e}{m}\cos\sigma_T \tag{49f}$$

$$a_{z_m} = \frac{\bar{q}S}{m}C_Z - \frac{F_e}{m}\sin\sigma_T \tag{49g}$$

with

$$C_X = \left(C_{L_0} + C_{L_V}\frac{V}{V_0} + C_{L_\alpha}\alpha\right)\sin\alpha - \left(C_{D_0} + C_{D_V}\frac{V}{V_0} + C_{D_\alpha}\alpha\right)\cos\alpha \tag{50a}$$

$$C_Z = -\left(C_{L_0} + C_{L_V}\frac{V}{V_0} + C_{L_\alpha}\alpha\right)\cos\alpha - \left(C_{D_0} + C_{D_V}\frac{V}{V_0} + C_{D_\alpha}\alpha\right)\sin\alpha \tag{50b}$$

The preceding system model now contains not only the usual trigonometric nonlinearities and those due to $\bar{q}$, but also those introduced by the transformations in Eqs. (50). Furthermore, inversions of the state variable V are required in the state equations. Thus, formulation of a model in wind-axis system to extract lift and drag derivatives leads to additional nonlinearities in the state equation for α and also in the observation equations for accelerations measured in body axes.

The unknown Θ for the above nonlinear model is

$$\Theta^T = [C_{D_0}, C_{D_V}, C_{D_\alpha}, C_{L_0}, C_{L_V}, C_{L_\alpha}, C_{m_0}, C_{m_V}, C_{m_\alpha}, C_{m_q}, C_{m_{\delta e}}, V_0, \alpha_0, \theta_0, q_0] \tag{51}$$

The same flight record analyzed using the previous model has been used again. The results obtained by accounting for measurement noise only are provided in Table 4. As in the last example (Fig. 4), differences in the measured and the responses estimated by the output error method were still discernible for the data analyzed.[25]

The estimation algorithms NLMLKL, MLEKF, and XAEKF are now used to directly estimate lift and drag derivatives using the preceding model accounting for both process and measurement noise. In all of these cases, no difficulties either of convergence or of filter divergence are encountered. The results of parameter estimation are summarized in Table 4. Time history plots of the measured and estimated responses obtained from all the algorithms were essentially the same as in Fig. 5. Figure 7 shows convergence of the lift and drag derivatives (augmented states) estimated using the extended Kalman filter, algorithm XAEKF.

The following general observations for the present example with a more complex system model can be made from the study of Table 4 and of Fig. 7.

1) The estimates of most parameters provided by the algorithms MLEKF and XAEKF are in reasonably good agreement.

2) The estimates of initial conditions θ_0 and q_0 in the body axis (Table 3) and in wind axis (Table 4) are in very good agreement. The estimate of true airspeed V_0 (wind-axis model) is 106.036 m/s, which agrees well with the results of model in body-axis, yielding $(u_0^2 + w_0^2)^{1/2} = 106.0385$. So, the estimate of $\alpha_0 = 0.1121$ rad with the transformed $\alpha_0 = \tan^{-1}(w_0/u_0) = 0.1122$ rad. Thus, axis transformation in the postulated model does not affect the estimates of the initial conditions, which is physically true.

3) The time per iteration required by the algorithm MLEKF for the model in wind axis is only slightly more than for the corresponding model in body axis. This is due to axis transformations included in the estimation model. However, this in-

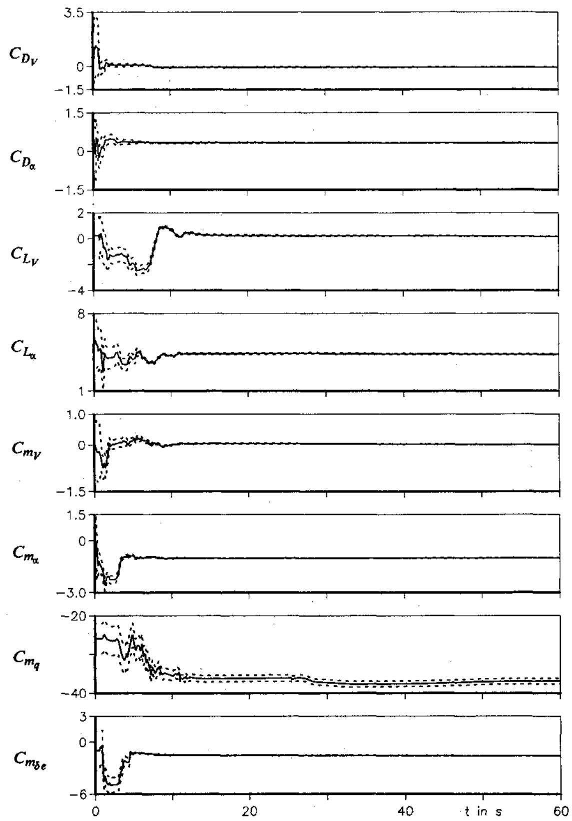

Fig. 7 Convergence of the nondimensional derivatives (wind axis) estimated using the extended Kalman filter: —, estimate; ---, estimate ± standard deviation.

Table 4 Estimation of aircraft longitudinal derivatives from flight-test data: nonlinear model with nondimensional derivatives (wind axis)

		ML estimates obtained by accounting for			
		Measurement noise only	Both process and measurement noise		
Parameter	Starting values	Algorithm NLMLKL	Algorithm NLMLKL	Algorithm MLEKF	Algorithm XAEKF
C_{D_V}	0.0077	−0.0640(5.1)[a]	−0.0645(3.9)	−0.0546(4.1)	−0.0552(4.1)
C_{D_α}	0.6742	0.3613(1.9)	0.3201(2.2)	0.3164(2.0)	0.3154(2.1)
C_{D_0}	0.0057	0.1128(3.2)	0.1227(2.4)	0.1079(2.5)	0.1087(2.5)
C_{L_V}	0.2603	0.1896(9.1)	0.1500(10)	0.1526(9.6)	0.1558(9.4)
C_{L_α}	5.3758	4.2632(1.0)	4.3386(1.1)	4.3029(1.0)	4.2971(1.0)
C_{L_0}	−0.3183	−0.1286(14)	−0.0952(20)	−0.0951(18)	−0.0978(17)
C_{m_V}	0.0189	−0.0226(20)	0.0022(152)	0.0080(37)	0.0079(37)
C_{m_α}	−0.4986	−1.0257(0.8)	−0.9705(1.1)	−0.9970(1.0)	−0.9965(1.0)
C_{m_q}	−25.844	−25.761(2.4)	−34.021(2.3)	−36.820(2.0)	−36.894(1.9)
$C_{m_{\delta_e}}$	−0.9907	−1.4988(1.0)	−1.5228(1.3)	−1.5891(1.1)	−1.5900(1.1)
C_{m_0}	0.0498	−0.1456(3.3)	0.1141(3.3)	0.1105(3.0)	0.1106(3.1)
V_0	105.977	105.948(0.1)	106.024(0.1)	106.036(0.04)	——
α_0	0.1971	0.1132(0.4)	0.1117(0.6)	0.1121(0.4)	——
θ_0	0.1317	0.0992(0.4)	0.1049(0.3)	0.1048(0.2)	——
q_0	0.0469	0.00028(303)	−0.00333(17)	−0.00349(5.8)	——
Iterations		7	6	3	——
CPU time, s		72	93	457	33

[a]Values in parentheses indicate standard deviations in %.

crease is only minor compared to the time required for computations of the state and response gradients.

4) For data containing process noise, the algorithm NLMLKL required fewer iterations than the output error method. The algorithm MLEKF with a time-varying filter requires even lesser iterations, but the computation time is significantly higher.

5) In the case of filtering approach, algorithm XAEKF, the estimates are found to converge within the first 20 s, as in the last example.

6) The average time per estimated parameter required by the algorithm NLMLKL is 93/15 = 6.2 s. It may be recalled that in this algorithm R is constrained to be diagonal. It has been found that relaxing the preceding constraint results in an increase of about 60% in the computational time required by NLMLKL. On the other hand, the algorithm XAEKF in which the filter implementation automatically leads to a full-size matrix of residual covariances, requires on an average 33/11 = 3s per parameter.

Concluding Remarks

A comparison of algorithms for parameter estimation using two different approaches, direct approach and filtering approach, has been made. Three model postulates, one linear and two nonlinear with different degrees of nonlinearities, as well as simulated and flight-test data have been used for this purpose. From this study, a general set of important conclusions, apart from those indicated for each example, can be summarized as follows:

1) The algorithms NLMLKL, MLEKF, and XAEKF provide sufficient flexibility to consider different nonlinear model postulates in parameter estimation with a minimum of problem-dependent modifications.

2) The accuracies of the various estimated parameters (standard deviations) obtained from the various algorithms are of the similar magnitudes.

3) For linear system models the algorithms KALML7 and NLMLKL, both using a steady-state filter for state estimation, are equivalent. The algorithm NLMLKL requires more computational time, which has to be accepted in light of the flexibility it provides in handling different model structures, which is essential for parameter estimation in nonlinear systems.

4) The algorithm MLEKF incorporates a time-varying filter within the iterative Gauss-Newton method for parameter update. Implementation of a time-varying filter, as carried out in this paper, requires explicit knowledge of both process and measurement noise matrices. Although it is feasible to estimate the process noise matrix using this algorithm, estimation of measurement noise matrix poses some problems. To overcome these difficulties, it will be necessary to implement the time-varying filter in terms of the covariance matrix of residuals.

5) The performance of the algorithm NLMLKL (Gauss-Newton method with a constant-gain filter) for the typical nonlinear examples considered has been very good. This algorithm is, in general, preferable to MLEKF, since the computational load is significantly lower. Furthermore, compared to algorithms MLEKF and XAEKF, it provides an option to estimate simultaneously the unknown noise covariances.

6) In the event that it becomes necessary to use a time-varying filter, the single-pass algorithm XAEKF (filtering approach), which consistently required much less time than the

iterative algorithm MLEKF (direct approach), appears to be a viable alternative.

7) For applications in which it is necessary to account for the full matrix of residual covariances, the computational time required by the algorithm XAEKF turns out to be less than or comparable to that required by the Gauss-Newton method with a steady-state filter (similar to NLMLKL). This appears to be contrary to the common notion that the nonlinear filtering approach to estimate the unknown system parameters requires more time due to the large number of resultant state variables.

8) To be able to use the filtering approach for completely unknown problems, it will be necessary to extend the filtering algorithm to include simultaneous estimation of noise covariances (adaptive filtering).

Acknowledgment

This work was partly sponsored by the Deutsche Forschungsgemeinschaft (DFG). Their support is gratefully acknowledged.

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